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# Bayesian New Two-Sided Group Chain Sampling Plan for Poisson Distribution with Gamma Prior

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**KEYWORDS**

*Bayesian; Acceptance Sampling; OC curve; Quality; Consumer's risk; Producer's risk; OC curve.*

**ABSTRACT**

Acceptance sampling is a vital approach used to determine whether a lot should be accepted or rejected based on a random sample drawn from that lot. This study introduces a Bayesian New Two-Sided Group Chain Sampling Plan (BNTSGChSP) that incorporates various combinations of design parameters. In this framework, inspections are conducted based on both preceding and succeeding lots. We model the acceptance probability (AP) of a lot using Poisson distribution for nonconforming and conforming products, with a Gamma distribution serving as an appropriate prior for the Poisson model. Our analysis identifies the inflection points for specified combinations of design parameters. The findings suggest that the BNTSGChSP offers a superior alternative to existing sampling plans for industrial practitioners.

## Introduction

Acceptance sampling is a crucial method for evaluating the quality of a lot and determining whether it meets established standards. The primary objective of acceptance sampling is to effectively differentiate between acceptable and unacceptable lots. Two main inspection techniques are employed: 100% inspection and sampling inspection. While 100% inspection provides a comprehensive assessment, it is often impractical due to time and cost constraints (Aziz et al., 2022; 2023). In contrast, sampling inspection is more realistic, efficient, and cost-effective. Under this approach, a lot is accepted or rejected based on the number of nonconforming items identified in a random sample (Montgomery 2009). Specifically, a lot is deemed acceptable unless the number of nonconforming units exceeds a predefined threshold, known as the maximum allowable quantity. This methodology serves as a vital tool in quality control, enabling organizations to maintain product standards while optimizing inspection resources.

Epstein (1954) proposed a single sampling plan (SSP) based on a lifetime distribution called exponential distribution of a submitted lot. Dodge (1955) proposed a chain-sampling strategy based on SSP that took into account multiple samples.

Bayesian sampling schemes necessitate the user defining the defect distribution from lot to lot. The expected distribution of product quality is the prior distribution of the sampling plan (Latha & Jeyabharathi 2014). The lot's decisiveness is determined by a combination of prior distribution and evidential skill based on

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sample information. Hald (1965) described a method for obtaining an SSP for attribute by minimising the average cost. Latha and Suresh (2002) addressed a Bayesian chain sampling plan (BChSP) for construction and performance measures using a gamma prior.

BChSP was extended by Hafeez and Aziz (2019), who proposed a Bayesian GChSP (BGChSP). More than one product can be inspected at the same time in BGChSP, depending on the number of testers available. In addition, when deciding on the current lot, this plan considers previous lots. They used a binomial distribution to calculate the average AP for a given percentage of nonconforming items. The plan was later extended for an average number of nonconforming by using the Poisson distribution as a prior distribution and the gamma distribution as a prior distribution (Hafeez, Aziz, Zain & Kamarudin 2022a). Based on BGChSP, Hafeez and Aziz (2022) developed and introduced a Bayesian two-sided GChSP (BTSGChSP). To make a decision for the current lot by considering preceding to succeeding lots. The average AP of lot was found to satisfy the consumer's and producer's risks that had been pre-specified.

Based on above mentioned plans, this study proposes a Bayesian new two-sided GChSP (BNTSGChSP) that considers preceding as well as succeeding lots. Poisson distribution function is used to estimate the AP of lot based on conforming and non-conforming products and gamma distribution is used as a suitable prior for Poisson distribution. Also, the plan indexed parameters for acceptable quality level (AQL) and limiting quality level (LQL) are designed. Four quality regions are found, namely: (i) quality decision region (QDR), (ii) probabilistic quality region (PQR), (iii) limiting quality region (LQR) and (iv) indifference quality region (IQR) for the specified values of the number of testers ($r$), shape parameter ($s$), preceding $i$ and succeeding $j$ lots. Also, numerical illustrations are provided for the parameters of prior distribution.

## Methodology

## Operating Procedure

The steps involve in the NTSGChSP operating procedure are as follows:

**i.** Select an ideal number of $g$ groups for each lot and assign $r$ items to each group which is the sample size $(n = g * r)$ required.

**ii.** Count the total number of nonconforming that are $d$ in current lot, $d_i$ in preceding $i$ lots and $d_j$ in succeeding $j$ lots.

**iii.** If more than one nonconforming is found in current lot $(d > 1)$, reject the lot.

**iv.** If $d = 0$ in the current sample, preceding $i$ and succeeding $j$ samples have at most one nonconforming $(d_i + d_j \leq 1)$, accept the lot.

**v.** If $d = 1$ in current sample and preceding $i$ and succeeding $j$ samples have no nonconforming $(d_i + d_j = 0)$, accept the lot.

Figure 1, represents a flow chart that summarises all of the above steps.

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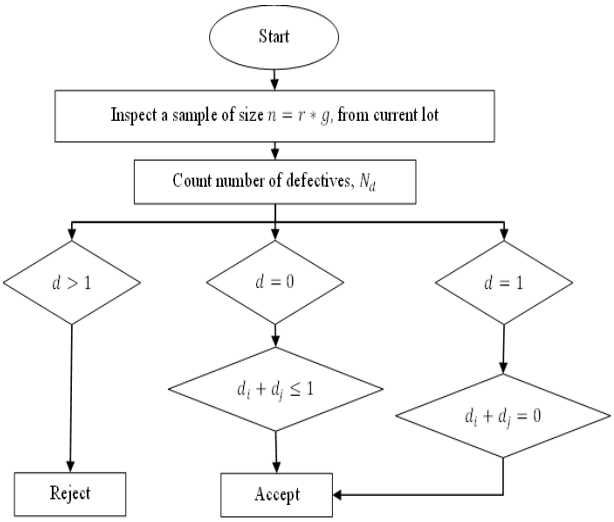


**Figure 1:** Operating procedure

For NTSGChSP, the above procedure can also be shown through a tree diagram for $i = j = 1$ in Figure 2, where $D$ denotes the nonconforming and $\overline{D}$ denotes conforming products.

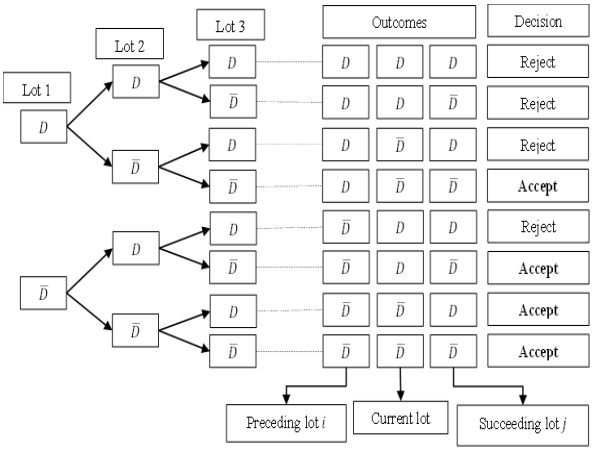


**Figure 1:** NTSGChSP tree diagram

It is clear from the tree diagram in Figure 2, that NTSGChSP has four acceptance criteria (AC). There are $\{D\overline{D}\overline{D}, \overline{D}D\overline{D}, \overline{D}\overline{D}D, \overline{D}\overline{D}\overline{D}\}$ possible outcomes that meet the acceptance criteria. To estimate the AP, the results can be written as probabilities (Kamal, Aziz & Zain 2019).

$$L(p) = P_1P_0P_0 + P_0P_1P_0 + P_0P_0P_1 + P_0P_0P_0; \quad (1)$$

$$L(p) = (P_0)^3 + 3P_1(P_0)^2. \quad (2)$$

For NTSGChSP, the general expression of the AP from Eq. (3) for $i = j = 1$ is:

$$L(p) = (P_0)^{i+j+1} + (i + j + 1)P_1(P_0)^{i+j}. \quad (3)$$

When developing the procedures, $L(p)$ can be calculated for the chain acceptance sampling plans, with the assumption that the underlying distribution for the plan is following either binomial or Poisson distribution (Rosaiah & Kantam 2005). For the average number of nonconforming, this paper considers Poisson distribution, such that:

$$p(c) = \frac{\mu^d}{d!}e^{-\mu}. \quad (4)$$

For group chain sampling, replace mean $\mu = np$ and $n = r * g$ in Poisson probability distribution function (PDF) and solve for $c = 0$ and $c = 1$. After solving Eq. (4), we obtain the probabilities of acceptance for zero and one nonconforming product:

$$P_0 = e^{-(r*g)p}; \quad (5)$$

$$P_1 = (r * g)\, p\, e^{-(r*g)p}. \quad (6)$$

Now in Eq. (3) we replace Eqs. (5) and (6), then we get:

$$L(p) = e^{-rgp(i+j+1)} + (i + j + 1)rgpe^{-rgp(i+j+1)} \quad (7)$$

For the equal number of preceding and succeeding lots $i = j$, Eq. (7) can be written as:

$$L(p) = e^{-rgp(2i+1)} + (2i + 1)rgpe^{-rgp(2i+1)}. \quad (8)$$

Consider the gamma distribution, which is a good prior for the Poisson distribution (Suresh & Sangeetha 2011):

$$f(p) = \frac{t^s}{\Gamma(s)} p^{s-1} e^{-tp}, \quad (9)$$

where $s > 0$, be the shape parameter and $t > 0$ rate parameter under the proposed sampling plan with mean $\mu = \frac{s}{t}$. The general expression used in Bayesian for the average AP of lot is (Hafeez, Aziz, Zain & Kamarudin 2022b; Hafeez, Aziz, & Du, 2023):

$$P = \int_0^\infty L(p) f(p)\, dp. \quad (10)$$

By replacing Eqs. (8) and (9) with Eq. (10) and after simplification we obtain:

$$P = \frac{t^s}{\Gamma(s)} \left[ \frac{\Gamma(s)}{(rg(2i+1)+t)^s} + rg(2i+1) \frac{\Gamma(s+1)}{(rg(2i+1)+t)^{s+1}} \right] \quad (11)$$

$$P = \left( \frac{t}{rg(2i+1)+t} \right)^s + \frac{rgst^s(2i+1)}{(rg(2i+1)+t)^{s+1}}. \quad (12)$$

Upon Replace mean $\mu = {}^{s}/_{t}$ that gives $t = {}^{s}/_{\mu}$ in Eq. (12) and simplifying:

$$P = \left( \frac{s}{rg\mu(2i+1)+s} \right)^s + rg\mu(2i+1) \left( \frac{s}{rg\mu(2i+1)+s} \right)^{s+1} \quad (13)$$

Now from Eq. (13), for $s = 1, 2, 3$, we get:

$$P = \frac{1}{rg\mu(2i+1)+1} + \frac{rg\mu(2i+1)}{(rg\mu(2i+1)+1)^2} \quad (14)$$

$$P = \frac{4}{(rg\mu(2i+1)+2)^2} + \frac{8rg\mu(2i+1)}{(rg\mu(2i+1)+2)^3} \quad (15)$$

$$P = \frac{27}{(rg\mu(2i+1)+3)^3} + \frac{81rg\mu(2i+1)}{(rg\mu(2i+1)+3)^4} \quad (16)$$

In Eqs. (14)-(16), Newton's approximation is used to estimate quality regions for BNTSGChSP, where $\mu$ is used as a point of control by reducing $P$ (Hafeez, Aziz & Du 2023). For the specified values of $s = 1, 2, 3$, $r = 2, 3, 4$ and $i = 1, 2, 3, 4$, Table 1 represents the average number of nonconforming.

**Table 1:** Average number of nonconforming for BNTSGChSP for certain parameters and $P$.

| *s* | *r* | *i* | 0.99 | 0.95 | 0.90 | 0.50 | 0.25 | 0.10 | 0.05 | 0.01 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 0.0185 | 0.048 | 0.0771 | 0.4024 | 1.0774 | 3.0811 | 6.4156 | 33.0831 |
| | | 2 | 0.0074 | 0.0192 | 0.0308 | 0.1609 | 0.4309 | 1.2325 | 2.5662 | 13.2333 |
| | | 3 | 0.004 | 0.0103 | 0.0165 | 0.0862 | 0.2309 | 0.6602 | 1.3748 | 7.0892 |
| | | 4 | 0.0093 | 0.024 | 0.0385 | 0.2012 | 0.5387 | 1.5406 | 3.2078 | 16.5416 |
| | 3 | 1 | 0.0037 | 0.0096 | 0.0154 | 0.0805 | 0.2154 | 0.6162 | 1.2831 | 6.6166 |
| | | 2 | 0.002 | 0.0051 | 0.0083 | 0.0431 | 0.1154 | 0.3301 | 0.6874 | 3.5446 |
| | | 3 | 0.0062 | 0.016 | 0.0257 | 0.1341 | 0.3591 | 1.027 | 2.1385 | 11.0277 |
| | | 4 | 0.0025 | 0.0064 | 0.0103 | 0.0537 | 0.1437 | 0.4108 | 0.8554 | 4.4111 |
| | 4 | 1 | 0.0013 | 0.0034 | 0.0055 | 0.0287 | 0.0769 | 0.2201 | 0.4583 | 2.3631 |
| | | 2 | 0.0047 | 0.012 | 0.0193 | 0.1006 | 0.2693 | 0.7703 | 1.6039 | 8.2708 |
| | | 3 | 0.0019 | 0.0048 | 0.0077 | 0.0402 | 0.1077 | 0.3081 | 0.6416 | 3.3083 |
| | | 4 | 0.001 | 0.0026 | 0.0041 | 0.0216 | 0.0577 | 0.165 | 0.3437 | 1.7723 |
| 2 | 2 | 1 | 0.0209 | 0.0522 | 0.0812 | 0.3333 | 0.6881 | 1.3691 | 2.1294 | 5.3256 |
| | | 2 | 0.0083 | 0.0208 | 0.0325 | 0.1333 | 0.2752 | 0.5476 | 0.8518 | 2.1303 |
| | | 3 | 0.0045 | 0.0112 | 0.0174 | 0.0714 | 0.1474 | 0.2934 | 0.4563 | 1.1412 |
| | | 4 | 0.0104 | 0.0261 | 0.0406 | 0.1667 | 0.344 | 0.6846 | 1.0647 | 2.6629 |
| | 3 | 1 | 0.0041 | 0.0104 | 0.0162 | 0.0667 | 0.1376 | 0.2738 | 0.4259 | 1.0651 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 0.0022 | 0.0056 | 0.0087 | 0.0357 | 0.0737 | 0.1467 | 0.2282 | 0.5706 |
| | | 3 | 0.007 | 0.0174 | 0.0271 | 0.1111 | 0.2294 | 0.4564 | 0.7098 | 1.7752 |
| | | 4 | 0.0028 | 0.0069 | 0.0108 | 0.0444 | 0.0918 | 0.1826 | 0.2839 | 0.7101 |
| | 4 | 1 | 0.0015 | 0.0037 | 0.0058 | 0.0238 | 0.0491 | 0.0978 | 0.1521 | 0.3804 |
| | | 2 | 0.0052 | 0.013 | 0.0203 | 0.0833 | 0.172 | 0.3423 | 0.5324 | 1.3314 |
| | | 3 | 0.0021 | 0.0052 | 0.0081 | 0.0333 | 0.0688 | 0.1369 | 0.2129 | 0.5326 |
| | | 4 | 0.0011 | 0.0028 | 0.0043 | 0.0179 | 0.0368 | 0.0733 | 0.1141 | 0.2853 |
| 3 | 2 | 1 | 0.0219 | 0.0541 | 0.0831 | 0.314 | 0.5957 | 1.0602 | 1.5112 | 3.0494 |
| | | 2 | 0.0088 | 0.0216 | 0.0333 | 0.1256 | 0.2383 | 0.4241 | 0.6045 | 1.2198 |
| | | 3 | 0.0047 | 0.0116 | 0.0178 | 0.0673 | 0.1276 | 0.2272 | 0.3238 | 0.6534 |
| | | 4 | 0.0109 | 0.0271 | 0.0416 | 0.157 | 0.2979 | 0.5301 | 0.7556 | 1.5247 |
| | 3 | 1 | 0.0044 | 0.0108 | 0.0166 | 0.0628 | 0.1191 | 0.2121 | 0.3022 | 0.6099 |
| | | 2 | 0.0024 | 0.0058 | 0.0089 | 0.0336 | 0.0638 | 0.1136 | 0.1619 | 0.3267 |
| | | 3 | 0.0073 | 0.018 | 0.0277 | 0.1047 | 0.1986 | 0.3534 | 0.5037 | 1.0165 |
| | | 4 | 0.0029 | 0.0072 | 0.0111 | 0.0419 | 0.0794 | 0.1414 | 0.2015 | 0.4066 |
| | 4 | 1 | 0.0016 | 0.0039 | 0.0059 | 0.0224 | 0.0425 | 0.0757 | 0.1079 | 0.2178 |
| | | 2 | 0.0055 | 0.0135 | 0.0208 | 0.0785 | 0.1489 | 0.2651 | 0.3778 | 0.7624 |
| | | 3 | 0.0022 | 0.0054 | 0.0083 | 0.0314 | 0.0596 | 0.106 | 0.1512 | 0.3049 |
| | | 4 | 0.0012 | 0.0029 | 0.0045 | 0.0168 | 0.0319 | 0.0568 | 0.081 | 0.1634 |

## Construction of Quality Regions

*Quality Decision Region (QDR)*

In this quality region, the product is accepted with the specified quality average by the engineer. Quality is reliably maintained up to $\mu_*$ LQL and a sudden decline in quality are expected. It is defined as $(\mu_1 < \mu < \mu_*)$ and denoted by $d_1 = \mu_* - \mu_1$ derived from the equation of APA, as given in Eq. (13).Therefore, gamma is prior distribution with the mean $\mu = {}^{s}/_{t}$ be the average quality of the product.

*Probabilistic Quality Region (PQR)*

In PQR the product is accepted with a minimum AP of 0.10 and a maximum AP of 0.95. PQR is defined as $(\mu_1 < \mu < \mu_2)$ and its range is denoted by $d_2 = \mu_2 - \mu_1$ derived from the equation of APA.

*Limiting Quality Region (LQR)*

The product is accepted with a minimum and maximum AP of 0.1 and 0.9. LQR is defined as an interval like $(\mu_* < \mu < \mu_2)$ and denoted by $d_3 = \mu_2 - \mu_*$. It is derived from the equation of the APA.

*Indifference Quality Region (IQR)*

In this quality region, the product is accepted with a minimum AP 0.50 and a maximum of 0.9. IQR is described as $(\mu_1 < \mu < \mu_0)$ and the range is denoted by $d_0 = \mu_0 - \mu_1$. It is derived from the equation of the APA.

## Selection of Sampling Plans

In Table 2, the ranges of QDR $(gd_1)$, PQR $(gd_2)$, LQR $(gd_3)$ and IQR $(gd_0)$, are shown with corresponding design parameters $s$, $r$ and $i$. The defined operating ratios $T = \frac{\mu_* - \mu_1}{\mu_2 - \mu_1} = \frac{g\mu_* - g\mu_1}{g\mu_2 - g\mu_1}$, $T_1 = \frac{\mu_* - \mu_1}{\mu_2 - \mu_*}$ and $T_2 = \frac{\mu_* - \mu_1}{\mu_0 - \mu_1}$, are used to characterize the sampling plan. For any given values of QDR $(d_1)$, PQR $(d_2)$, LQR $(d_3)$ and IQR $(d_0)$, we can find the operating ratios $T = \frac{d_1}{d_2}$, $T_1 = \frac{d_1}{d_3}$ and $T_2 = \frac{d_1}{d_0}$. Find the value parallel to the design parameters $s$, $r$ and $i$ which is close to the specified ratio under the column

of $T$, $T_1$ and $T_2$ in Table 2. From this ratio, we can determine the minimum number of groups $g$ and other design parameters for the BNTSGChSP.

## Numerical Examples

Given that $\mu_1 = 0.01$, $r = 2$, $s = 2$ and $i = 4$, compute the respective values of QDR, PQR, LQR, IQR, $T$, $T_1$ and $T_2$ from Table 2. The corresponding values are $gd_1 = 0.0145$, $gd_2 = 0.6585$, $gd_3 = 0.644$, $gd_0 = 0.1406$ and the ratios $T = 0.02198$, $T_1 = 0.02247$, $T_2 = 0.10295$. From Table 1, the corresponding value of $g\mu_1 = 0.0261$ from which the required minimum number of groups can be obtained: $g = {}^{g\mu_1}/_{\mu_1} = {}^{0.0261}/_{0.01} = 6.21 \cong 7$. Thus, the selected parameters for BNTSGChSP are $r = 2$, $s = 2$ and $i = 4$ with a minimum number of groups $g = 7$. Also, the values of QDR $d_1 = 0.00207$, PQR $d_2 = 0.0941$, LQR $d_3 = 0.092$, IQR $d_0 = 0.0201$, and the ratios $T = 0.02198$, $T_1 = 0.02247$, $T_2 = 0.10295$.

**Table 2:** For specified $s, r$ and $i$ values of QDR, PQR, LQR, IQR and operating ratios.

| $s$ | $r$ | $i$ | $g\mu_1$ | $g\mu_*$ | $g\mu_0$ | $g\mu_2$ | $gd_1$ | $gd_2$ | $gd_3$ | $gd_0$ | $T$ | $T_1$ | $T_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 0.048 | 0.0771 | 0.4024 | 3.0811 | 0.0291 | 3.0331 | 3.0041 | 0.3544 | 0.00958 | 0.00968 | 0.08204 |
| | | 2 | 0.0192 | 0.0308 | 0.1609 | 1.2325 | 0.0116 | 1.2133 | 1.2016 | 0.1417 | 0.00958 | 0.00968 | 0.08204 |
| | | 3 | 0.0103 | 0.0165 | 0.0862 | 0.6602 | 0.0062 | 0.6499 | 0.6437 | 0.0759 | 0.00956 | 0.00965 | 0.08186 |
| | | 4 | 0.024 | 0.0385 | 0.2012 | 1.5406 | 0.0145 | 1.5166 | 1.502 | 0.1772 | 0.00959 | 0.00968 | 0.08207 |
| | 3 | 1 | 0.0096 | 0.0154 | 0.0805 | 0.6162 | 0.0058 | 0.6066 | 0.6008 | 0.0709 | 0.00957 | 0.00966 | 0.08191 |
| | | 2 | 0.0051 | 0.0083 | 0.0431 | 0.3301 | 0.0031 | 0.325 | 0.3219 | 0.038 | 0.00961 | 0.00971 | 0.08231 |
| | | 3 | 0.016 | 0.0257 | 0.1341 | 1.027 | 0.0097 | 1.011 | 1.0014 | 0.1181 | 0.00959 | 0.00968 | 0.08205 |
| | | 4 | 0.0064 | 0.0103 | 0.0537 | 0.4108 | 0.0039 | 0.4044 | 0.4006 | 0.0472 | 0.00958 | 0.00968 | 0.08204 |
| | 4 | 1 | 0.0034 | 0.0055 | 0.0287 | 0.2201 | 0.0021 | 0.2166 | 0.2146 | 0.0253 | 0.00956 | 0.00965 | 0.08182 |
| | | 2 | 0.012 | 0.0193 | 0.1006 | 0.7703 | 0.0072 | 0.7583 | 0.751 | 0.0885 | 0.00955 | 0.00964 | 0.0818 |
| | | 3 | 0.0048 | 0.0077 | 0.0402 | 0.3081 | 0.0029 | 0.3033 | 0.3004 | 0.0354 | 0.00956 | 0.00965 | 0.08186 |
| | | 4 | 0.0026 | 0.0041 | 0.0216 | 0.165 | 0.0015 | 0.1625 | 0.1609 | 0.019 | 0.00948 | 0.00958 | 0.08123 |
| 2 | 2 | 1 | 0.0522 | 0.0812 | 0.3333 | 1.3691 | 0.029 | 1.3169 | 1.2879 | 0.2812 | 0.02201 | 0.0225 | 0.10308 |
| | | 2 | 0.0208 | 0.0325 | 0.1333 | 0.5476 | 0.0116 | 0.5268 | 0.5152 | 0.1125 | 0.02205 | 0.02254 | 0.10324 |
| | | 3 | 0.0112 | 0.0174 | 0.0714 | 0.2934 | 0.0062 | 0.2822 | 0.276 | 0.0602 | 0.02199 | 0.02249 | 0.10301 |
| | | 4 | 0.0261 | 0.0406 | 0.1667 | 0.6846 | 0.0145 | 0.6585 | 0.644 | 0.1406 | 0.02198 | 0.02247 | 0.10295 |
| | 3 | 1 | 0.0104 | 0.0162 | 0.0667 | 0.2738 | 0.0058 | 0.2634 | 0.2576 | 0.0562 | 0.022 | 0.0225 | 0.10305 |
| | | 2 | 0.0056 | 0.0087 | 0.0357 | 0.1467 | 0.0031 | 0.1411 | 0.138 | 0.0301 | 0.02207 | 0.02257 | 0.10334 |
| | | 3 | 0.0174 | 0.0271 | 0.1111 | 0.4564 | 0.0097 | 0.439 | 0.4293 | 0.0937 | 0.02202 | 0.02251 | 0.10312 |
| | | 4 | 0.0069 | 0.0108 | 0.0444 | 0.1826 | 0.0039 | 0.1756 | 0.1718 | 0.0375 | 0.02204 | 0.02254 | 0.10319 |
| | 4 | 1 | 0.0037 | 0.0058 | 0.0238 | 0.0978 | 0.0021 | 0.0941 | 0.092 | 0.0201 | 0.02197 | 0.02247 | 0.1029 |
| | | 2 | 0.013 | 0.0203 | 0.0833 | 0.3423 | 0.0072 | 0.3292 | 0.322 | 0.0703 | 0.02195 | 0.02244 | 0.1028 |
| | | 3 | 0.0052 | 0.0081 | 0.0333 | 0.1369 | 0.0029 | 0.1317 | 0.1288 | 0.0281 | 0.02208 | 0.02258 | 0.10344 |
| | | 4 | 0.0028 | 0.0043 | 0.0179 | 0.0733 | 0.0015 | 0.0705 | 0.069 | 0.0151 | 0.02187 | 0.02236 | 0.10232 |
| 3 | 2 | 1 | 0.0541 | 0.0831 | 0.314 | 1.0602 | 0.0291 | 1.0062 | 0.9771 | 0.2599 | 0.02888 | 0.02974 | 0.11183 |
| | | 2 | 0.0216 | 0.0333 | 0.1256 | 0.4241 | 0.0117 | 0.4025 | 0.3908 | 0.104 | 0.02896 | 0.02982 | 0.11208 |
| | | 3 | 0.0116 | 0.0178 | 0.0673 | 0.2272 | 0.0063 | 0.2156 | 0.2094 | 0.0557 | 0.029 | 0.02986 | 0.11227 |
| | | 4 | 0.0271 | 0.0416 | 0.157 | 0.5301 | 0.0145 | 0.5031 | 0.4885 | 0.1299 | 0.02887 | 0.02973 | 0.11178 |
| | 3 | 1 | 0.0108 | 0.0166 | 0.0628 | 0.2121 | 0.0058 | 0.2012 | 0.1954 | 0.052 | 0.02877 | 0.02962 | 0.11137 |
| | | 2 | 0.0058 | 0.0089 | 0.0336 | 0.1136 | 0.0032 | 0.1078 | 0.1047 | 0.0279 | 0.02924 | 0.03012 | 0.11319 |
| | | 3 | 0.018 | 0.0277 | 0.1047 | 0.3534 | 0.0097 | 0.3354 | 0.3257 | 0.0866 | 0.02892 | 0.02978 | 0.11194 |
| | | 4 | 0.0072 | 0.0111 | 0.0419 | 0.1414 | 0.0039 | 0.1342 | 0.1303 | 0.0347 | 0.02883 | 0.02968 | 0.11161 |
| | 4 | 1 | 0.0039 | 0.0059 | 0.0224 | 0.0757 | 0.002 | 0.0718 | 0.0698 | 0.0185 | 0.02823 | 0.02905 | 0.10942 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 0.0135 | 0.0208 | 0.0785 | 0.2651 | 0.0073 | 0.2515 | 0.2443 | 0.065 | 0.02887 | 0.02973 | 0.11179 |
| 3 | 0.0054 | 0.0083 | 0.0314 | 0.106 | 0.0029 | 0.1007 | 0.0977 | 0.026 | 0.02926 | 0.03014 | 0.11319 |
| 4 | 0.0029 | 0.0045 | 0.0168 | 0.0568 | 0.0015 | 0.0539 | 0.0523 | 0.0139 | 0.02842 | 0.02925 | 0.10993 |

*For Specified QDR and PQR*

When QDR and PQR are specified, then Table 2 is used to construct the plan for any values of $d_1$ and $d_2$ we can find the ratio $T = {d_1}/{d_2}$ which is monotonic increasing function. Find the value which is approximately equal to the specified ratio under column $T$ in Table 2 and note the corresponding values of $s$, $r$ and $i$. By this procedure, we can find all parameter values for BNTSGChSP with a minimum number of groups $g$.

Suppose a manufacturing company required QDR $d_1 = 0.002$ and PQR $d_2 = 0.07$, then the calculated operating ratio is $T = 0.02857$. From Table 2, closest value is observed to be $T = 0.02842$, with parallel values of $s = 3$, $r = 4$ and $i = 4$. So, for this operating ratio $gd_1 = 0.0015$ and $gd_2 = 0.0539$, then the value of $g = {gd_1}/{d_1} = {0.0015}/{0.002} = 0.75 \cong 1$.
Hence for the required QDR $d_1 = 0.002$ and PQR $d_2 = 0.07$ design parameters of BNTSGChSP are $s = 3$, $r = 4$ and $i = 4$ with a minimum number of groups $g = 1$.

*For Specified QDR and LQR*

Let in a manufacturer company required QDR $d_1 = 0.002$ and LQR $d_3 = 0.09$, then the calculated operating ratio is $T_1 = 0.0222$. From Table 2, the nearest value is found to be $T_1 = 0.02236$, with corresponding values of design parameters $s = 2$, $r = 4$ and $i = 4$. So, for this operating ratio $gd_1 = 0.0015$ and $gd_3 = 0.069$, then the value of $g = {gd_1}/{d_1} = {0.0015}/{0.002} = 0.75 \cong 1$. Hence for the required QDR $d_1 = 0.002$ and LQR $d_3 = 0.09$ design parameters of BNTSGChSP are $s = 2$, $r = 4$ and $i = 4$ with a minimum number of groups $g = 1$.

*For Specified QDR and IQR*

Let in a manufacturer company required QDR $d_1 = 0.01$ and IQR $d_0 = 0.09$, then the calculated operating ratio is $T_2 = 0.1111$. From Table 2, the nearest value is found to be $T_2 = 0.11161$, with parallel values of $s = 3$, $r = 3$ and $i = 4$. So, for this operating ratio $gd_1 = 0.0039$ and $gd_0 = 0.0347$, then the value of $g = {gd_1}/{d_1} = {0.0039}/{0.01} = 0.39 \cong 1$. Hence for the required QDR $d_1 = 0.01$ and LQR $d_0 = 0.09$ design parameters of BTSGChSP are $s = 3$, $r = 3$ and $i = 4$ with a minimum number of groups $g = 1$..

**Performance comparison**

Consider $s = 2, i = j = 3$, then the OC curves for $r = 2, 3, 4$, are displayed in Figure 3.

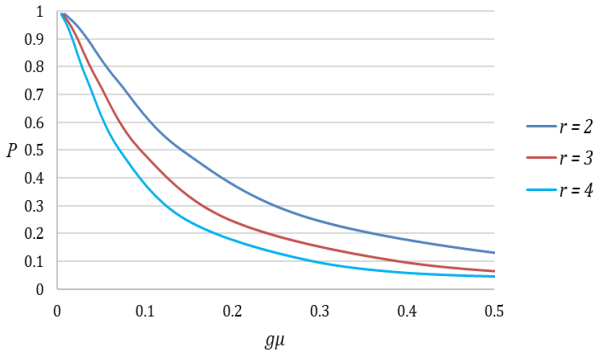


**Figure 3:** OC curves for $r = 2, 3, 4$

When $r = 4, i = j = 3$, then OC curves for shape parameter values $s = 1, 2, 3$ are shown in Figure 4.

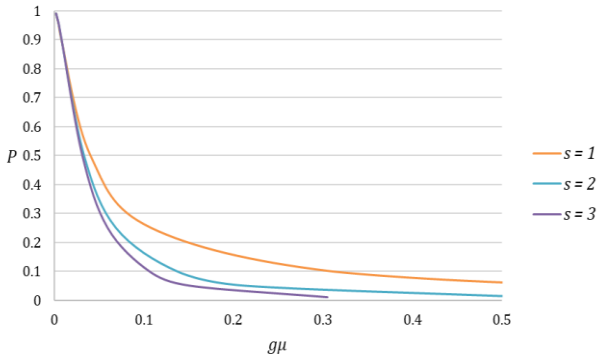


**Figure 4:** OC curves for $s = 1, 2, 3$

From Figs. 3 and 4, we can conclude that the ideal OC curve can be achieved by increasing the value of preceding or succeeding lots, shape parameter and the number of testers.

For comparison purposes, BNTSGChSP is compared with existing BGChSP (Hafeez & Aziz 2019) for the same design parameter values. For $s = 2, r = 3$ and $i = j = 2$, the average number of nonconformings is plotted against the average AP in Figure 5.

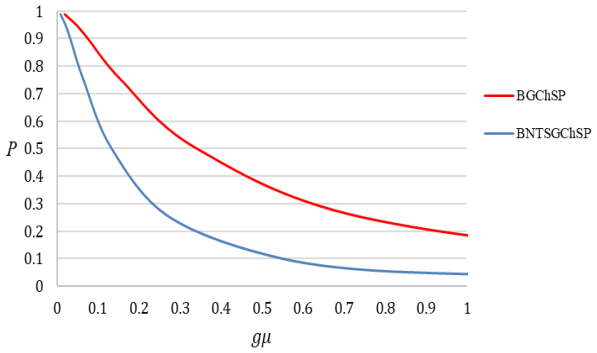


**Figure 5:** OC curves for BGChSP and BNTSGChSP

From Figure 5, it can conclude that the BNTSGChSP OC curve is more ideal than the existing BGChSP (Hafeez & Aziz 2019). For both plans, if the values of all design parameters are the same, BNTSGChSP gives a smaller number of nonconforming than BGChSP.

## Conclusion

This study focuses on the BNTSGChSP, estimating four quality regions based on specified producer's and consumer's risks. This plan effectively safeguards the interests of both producers and consumers, providing a balanced approach to quality assurance. The proposed BNTSGChSP is applicable for evaluating a range of electronic components, including transportation electronics systems and other critical applications. Future research could expand upon this work by exploring additional distributions and integrating various quality and reliability characteristics, thereby broadening the scope and applicability of the sampling plan.

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